Relationship between Traffic Density, Speed and Safety and Its Implication on Setting Variable Speed Limits on Freeways

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ABSTRACT

Speed-flow relationships for a typical basic freeway segment are well understood at present and are documented by the successive editions of the Highway Capacity Manual. All recent freeway studies show that speed on freeways is insensitive to flow in the low to mid range. Increase in flow and density without reduction in speed has a significant influence on safety. Constructive discussion of this influence, however, is largely absent from extant literature. Empirical examination of the relationship between flow/density, speed and crash rate on selected freeways in Colorado suggests that as flow/density increases crash rate initially remains constant until a certain critical threshold combination of speed and density is reached. Once this threshold is exceeded the crash rate rapidly rises. The rise in crash rate may possibly be explained by the fact that compression of flow without notable reduction in speed produces headways so small that it becomes very difficult or impossible to compensate for driver's error to avoid a crash. In addition to calibrating corridor specific SPFs relating crash rate to hourly volume/density and speed this paper proposes a variable speed limit (VSL) algorithm intended to slow traffic down in real time in advance of a high speed-high density operational regime. Deployment of such an algorithm has the potential to improve safety and reduce travel time variability.

"We are to admit no more causes of natural things than such
as are both true and sufficient to explain their appearances."

Isaac Newton

INTRODUCTION

Speed-flow and density-flow relationships for a typical basic freeway segment are well understood at present and are documented by the successive editions of the Highway Capacity Manual (HCM)(1). All recent freeway studies show that speed on freeways is insensitive to flow in the low to mid range. Increase in flow and density without notable reduction in speed has a significant influence on safety, this influence, however, has not been studied extensively and has attracted only limited interest from researchers to date. Lord et al. (2) observed that most of research has focused on determining the relationship between crashes and annual average daily traffic (AADT), while little attention has been focused on the relationships of vehicle density, level of service (LOS), vehicle occupancy, volume to capacity (V/C) ratio and speed distribution. Zhou and Sisiopiku (3) found that crash rates typically follow a U-shaped relationship when plotted as a function of V/C ratio. Traditional safety performance functions relate accident occurrence to average annual daily traffic (AADT). Persaud and Dzbik (4) observed that a difficulty with this approach is that a freeway with intense flow during rush periods would clearly have a different accident potential than a freeway with the same AADT but with flow evenly spread out throughout the day. Kononov et al. (5) observed that on uncongested freeways the number of crashes increases moderately with increase in traffic; however, once some critical traffic density is reached, the number of crashes begins to increase at a much faster rate with an increase in traffic. Garber and Subramanyan (6) related crashes to lane occupancy and concluded that peak crash rates do not occur during peak flows. Harwood in (7) noted that it would be extremely valuable to know how safety varies with Volume/Capacity (V/C) ratio and what V/C ratios provide the minimum accident rate. Hall and Pendelton (8) observed that knowledge of the definite relationship between V/C ratio and crash rate would help engineers and planners asses safety implications of highway improvements designed to increase capacity. In (2) Lord et al. conclude that "despite overall progress, there is still no clear understanding about the effects of different traffic flow characteristics on safety."

Figure 1 (EXHIBIT 23-3) from the 2000 Edition of the Highway Capacity Manual (HCM) (1) shows the speed-volume/density relationship and Level of Service (LOS) for basic freeway segments. It reflects the fact that drivers on modern freeways are slowing down very little or not at all as LOS deteriorates from A to D. Considering that perception-reaction time and vehicle characteristics remain unchanged while there are considerably more vehicles in the same space traveling at substantially the same speed as before, an increased probability of crash occurrence is highly plausible. This increase would be reflected by changes in the crash rate. For instance on a freeway with free-flow speed of 70 mph at point 1 carrying 600 pc/h/ln (V_1) has density d_1 = 8.6 pc/mi/ln and operates at LOS A. When congestion builds up to 1750 pc/mi/ln (V_2) (boundary between LOS-C and LOS-D.) the resulting density rises to d_2 = 26 pc/mi/ln and operating speed drops only slightly to 68 mph.

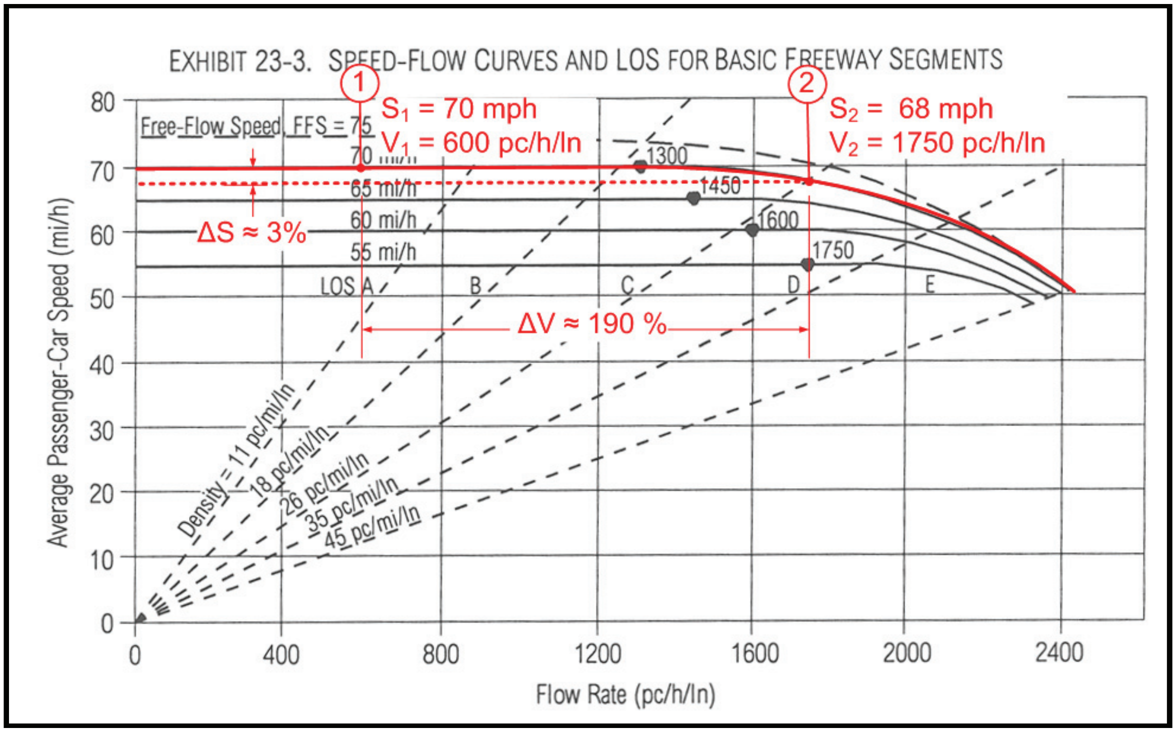

Figure 1 Speed-Flow Curves and LOS

As a transition is made from point 1 to point 2 we observe densities that are almost 3 times greater and a decrease in speed of only 3%. When these flow parameters are examined for a freeway with Free-Flow Speed of 55 mph we observe that volume rises from 600 vph (density =10.9 pc/mi/ln) to 1,750 vph (density=31.8 pc/mi/ln) without any speed reduction. Compression of flow without corresponding reduction in speed is likely to have an adverse effect on safety; calibration of this effect is the focus of this paper. Additionally use of Variable Speed Limits to mitigate this problem is explored.

MODEL DEVELOPMENT

Dataset Preparation

Hourly volume, operating speed and free-flow speed data were collected from existing automatic traffic recording (ATR) stations around the Denver metropolitan area 4-lane freeways and a segment of Interstate 70 (I-70), which carries ski resort traffic in mountainous terrain. Mainline crash history was obtained from the CDOT crash database for every hour over a five (5) year period (2001-2006) for every freeway in the dataset. All crashes that occurred on ramps and cross roads were removed prior to fitting the models.

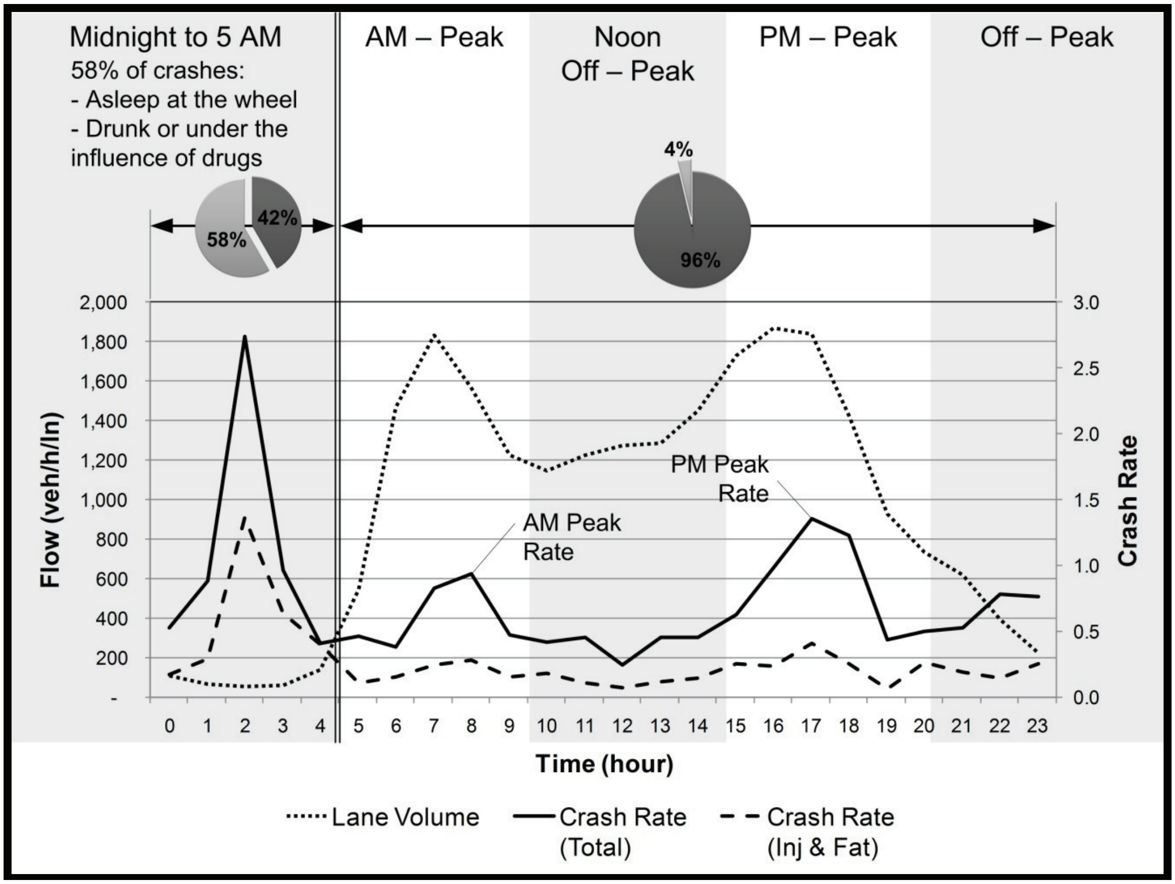

Figure 2 Changes in Volume and Crash Rate over the 24 hr Period on Denver Area Urban Freeways

Matching hourly volume on every segment with its crash history enabled us to compute crash rate for every hour of the 24 hour period for all freeways in the dataset. A graph representing typical Denver area 4-lane freeways demonstrating changes in volume and crash rates throughout the day is presented in Figure 2.

It is of interest to note that between the hours of midnight and 5 AM nearly 60% of all crashes involved alcohol or drug use or falling asleep at the wheel as compared with only 4% the rest of the day. Such a dramatic difference in driver performance abilities and crash causality suggests a qualitatively different phenomenon. A mix of impaired and fatigued drivers with low volumes produces very high crash rates when compared with day time safety performance of the same segments. It may possibly explain the U-shaped relationship identified by Zhou and Sisiopiku in (3). The impaired driver issue, a largely behavioral problem, is distinct from issues near or at peak times. Recognizing this, a portion of the dataset containing safety performance data between midnight and 5 AM was removed prior to calibration of the corridor specific Safety Performance Functions. Additionally Figure 2 suggests that the afternoon peak is characterized by higher crash rates than the morning peak. It may possibly be speculated that commuters are more fatigued; less focused on the driving task and are more eager to get home from work. Also it may possibly be attributed to more secondary crashes which result from the longer duration of the PM peak period. With this in mind we have calibrated separate corridor specific Safety Performance Functions (SPF) containing morning and afternoon peak periods on urban freeways and seasonal safety performance function of I-70 carrying ski resort traffic.

Relating Basic Kinematics with Flow Theory

A possible way to explore the relationship between safety and traffic flow parameters is to examine average distance between vehicles available at different combinations of density and speed and to compare it to the distance required to slow down in order to avoid a crash due to sudden change in traffic flow conditions or driver's error. Average distance between vehicles can be approximately expressed as a function of density.

$$h_i = c\frac{1}{d_i} \quad (1)$$

Where

h_i – Average distance between cars under operational conditions i

d_i – Density (pcpmpl) under operational conditions i

c – Constant which approximately accounts for distance taken up by vehicles

According to the basic motion equation for deceleration

$$D_r = \frac{S_i^2 - S_e^2}{2a} \quad (2)$$

Where

D_r - Distance required to decelerate from S_i to S_e

S_i - Initial Speed

S_e- End Speed

a - Rate of deceleration (assumed constant)

Under safe operational conditions, the distance required to slow down to avoid a crash has to be less than average available distance between vehicles, therefore

$$\frac{S_i^2 - S_e^2}{2a} < h_i = c\frac{1}{d_i} \quad (3)$$

Applying equation (3) to "back of the queue scenario" frequently encountered on the freeways where S_e=0 equation (3) becomes:

$$\frac{S_i^2}{2a} < c\frac{1}{d_i} \quad (4)$$

This can now be modified as follows:

$$d_i S_i^2 < c2a \quad (5)$$

The right side of the equation can be viewed as a constant $C_{0,}$ and therefore the equation becomes the threshold inequality below:

$$d_i S^2 < C_0 \quad (6)$$

Another possible scenario may involve a sudden need to decelerate due to a slower moving vehicle ahead. The time $t_{i\text{-}e}$ required to decelerate (at an assumed constant rate) from S_i to S_e satisfies the following basic kinematics equation:

$$t_{i-e} = \frac{S_i - S_e}{a} \quad (7)$$

During time $t_{i\text{-}e}$ slower moving vehicle traveling at speed S_e will travel the distance D_e which can be expressed as so:

$$D_e = S_e t_{i-e} = \frac{S_e(S_i - S_e)}{a} \quad (8)$$

In the process of deceleration S_e can be expressed as some proportion *p* of S_i

$$S_e = pS_i \quad (9)$$

Substituting S_e from equation (9) into equation (8) the following expression is obtained:

$$D_e = \frac{pS_i(S_i - pS_i)}{a} = \frac{pS_i{}^2 - p^2 S_i^2}{a} = \frac{pS_i{}^2(1-p)}{a} \quad (10)$$

As the faster moving vehicle decelerates from S_i to S_e it will travel distance D_r described by equation (2)

$$D_r = \frac{S_i^2 - S_e^2}{2a} \quad (2)$$

Replacing Se with pS_i distance D_r can now be expressed as follows:

$$D_r = \frac{S_i^2 - p^2 S_i^2}{2a} = \frac{S_i^2(1-p^2)}{2a} = \frac{S_i^2(1-p)(1+p)}{2a} \quad (11)$$

A relative change in distance Δ between two vehicles over the time of deceleration from S_i to S_e is computed below:

$$\Delta = \frac{S_i^2(1-p)(1+p)}{2a} - \frac{2pS_i{}^2(1-p)}{2a} = \frac{S_i^2(1-p)(1+p-2p)}{2(a)} = \frac{S_i^2(1-p)^2}{2a} \quad (12)$$

Requiring that Δ be less than some multiple c_1 of the average distance between vehicles h_i produces the threshold inequality below

$$\frac{S_i^2(1-p)^2}{2a} < c_1\, h_i \;= c_2\,\frac{1}{d_1} \text{ or } \quad (13)$$

$$d_i S_i^2 < C \quad (14)$$

where c_1, c_2 , p, a and C are constant with respect to speed and volume.

Comparing available distance between cars traveling at speed S_i with requisite distance to avoid a crash via dS^2 does not address all modes of crash occurrence. This model represents only a simplified version of reality. However, considering that over 70% of freeway crashes are rear-ends and sideswipes it addresses the most prevalent mechanisms of crash occurrence. The appearance of density and speed terms in the inequality above motivates us to consider density in concert with speed as we explore the relationship between flow characteristics and safety using Neural Networks. In particular, it suggests that properties beyond volume V=dS, should be considered. Using V alone runs counter to the expectation that a segment with high volume produced by high density at low speed may have a different crash rate than the same segment with the same volume a produced by, say, half the density and twice the speed. The discussion that follows uses the form of the threshold inequality derived above. This form should be verified or modified based on additional empirical evidence.

Neural Networks

Corridor specific SPFs relating freeway flow parameters with crash rate were developed using Neural Networks, that is a subset of a general class of nonlinear models. We used Neural Networks to analyze the data which consists of observed, univariate responses Y_i known to be dependent on corresponding one-dimensional inputs x_i. Neural Networks are not constrained by a pre-selected functional form and specific distributional assumptions. For our application, Y_i = Crash Rate (acc/mvmt) and x_i = dS^2, where d is density (pcpmpl) and S is speed (mph). The model becomes:

$$Y_i = f(x_i, \theta) + e_i$$

where,

$f(x_i, \theta)$ = the nonlinear function relating Y_i to the independent variable x_i for the *ith* observational unit,

θ = a *p*-dimensional vector of unknown parameters, and

e_i = is a sequence of independent random variables.

The goal of the nonlinear regression analysis is to find the function f that best reproduces the observed data. A form of the response function used in many engineering applications is a feed forward neural network model with a single layer of hidden units. The form of the model is:

$$f(x,\theta)=\beta_0+\sum_{k=1}^{K}\beta_k\varphi(x\ \gamma_k+\mu_k)$$

where,

$\varphi(u)=e^u/(1+e^u)$- a logistic distribution function
β_k = are known as connection weights and
$\beta_0,\beta_1,\gamma_i,\mu_i,i=1,...K$ = the parameters to be estimated
μ_k = the biases , Ripley (9).
K = the number of hidden units

The function f is a very flexible nonlinear model used in this application to capture the overall shape of the observed data. The function $\varphi(u)$ is a logistic distribution function. When $K=1$, there is one hidden unit. In this case, the function performs a linear transformation of the input *x* and then applies the logistic function $\varphi(u)$, followed by another linear transformation. The result is still a very flexible nonlinear model.

The parameters $\beta_0,\beta_1,\gamma_i,\mu_1$ for each dataset are unknown and will be estimated by nonlinear least squares. The complexity for this application is the number of hidden units K in the model. We have chosen $K=1$ based on general understanding of the underlying physical phenomenon. Additionally the complexity of the model is most often chosen based on the generalized cross validation (GCV) model-selection criterion. Cross-validation is a standard approach for selecting smoothing parameters in nonparametric regression described by Wahba (10). Overall model fit to the data is quite good (Figures 3-7).

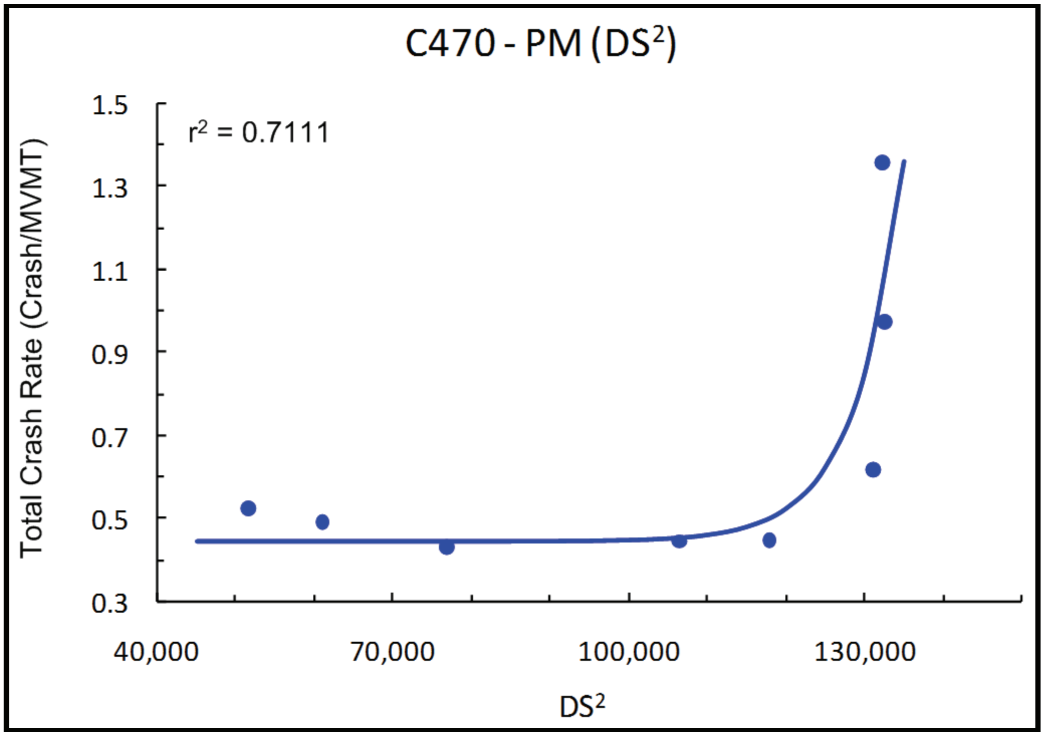

Figure 3 Corridor Specific SPF C-470 (PM) (4-lanes, 7 miles)

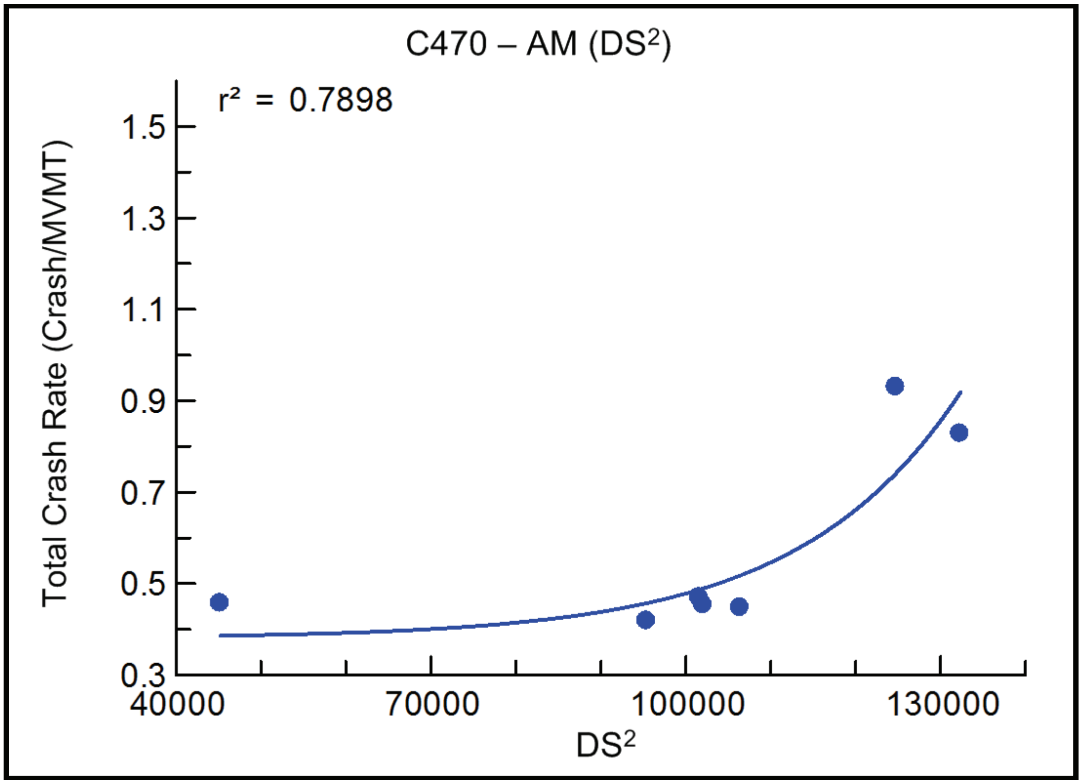

Figure 4 Corridor Specific SPF C-470 (AM) (4 lanes, 7 miles)

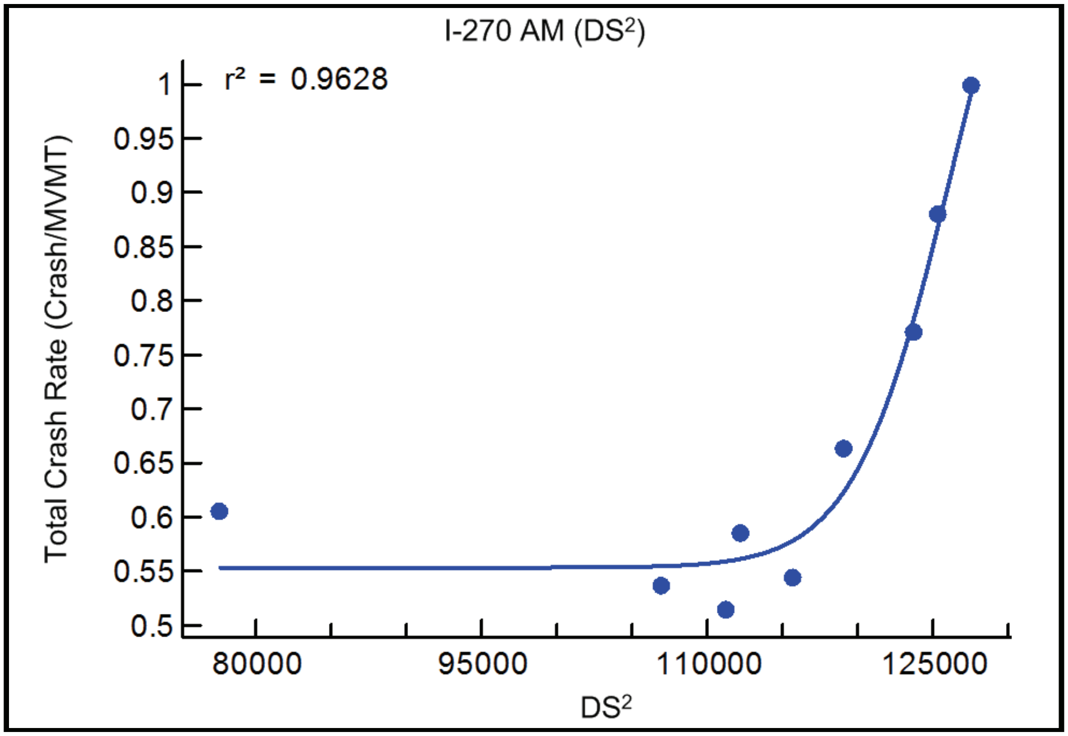

Figure 5 Corridor Specific SPF I-270 (AM), (4 lanes, 5 miles)

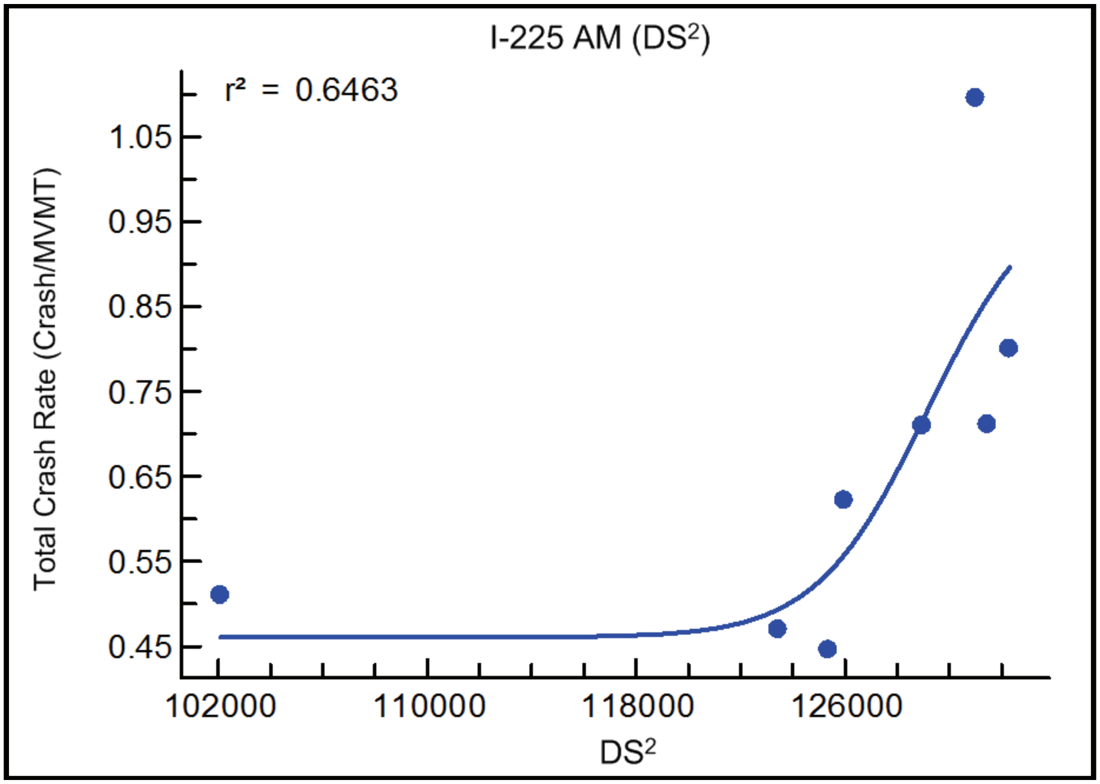

Figure 6 Corridor Specific SPF I-225 (AM) (4 lanes, 6 miles)

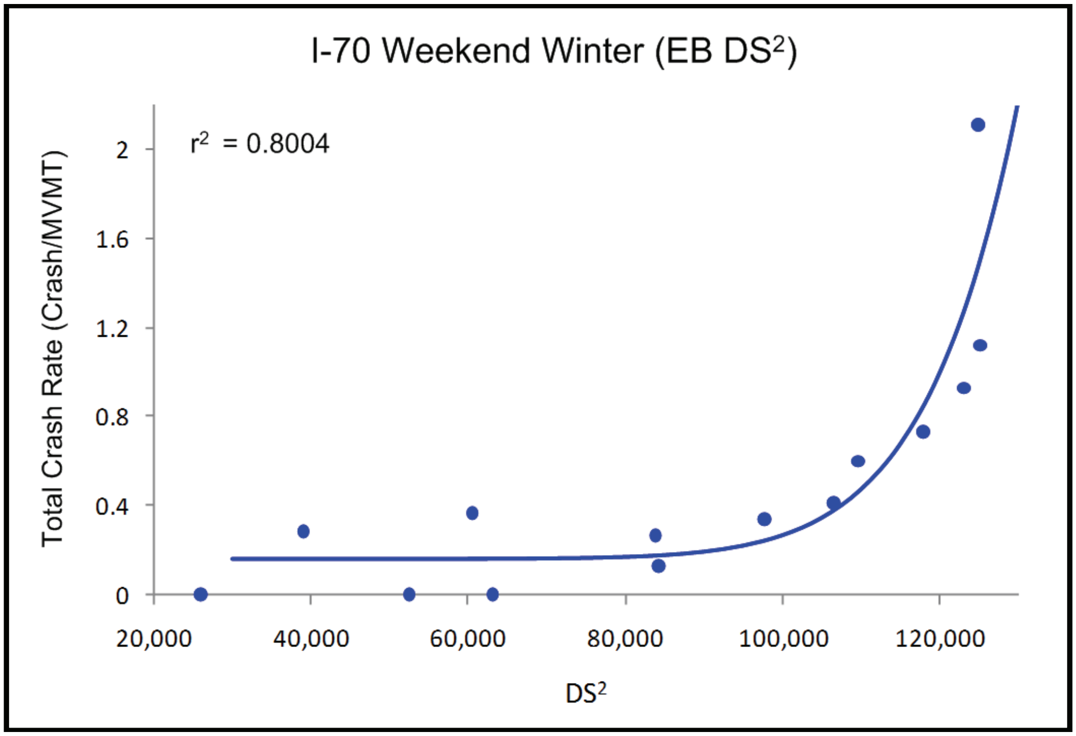

Figure 7 Corridor Specific SPF I-70 EB Winter Season (PM) (4 lanes, 5 miles)

The product of traffic density (d_i) times its speed squared (S_i^2) as an explanatory variable enables us to consider density in concert with speed as we examine the relationship between flow characteristics and safety. Figures 3-7 reflect these relationships for several freeways in the Denver metro area and a heavily traveled rural freeway in a mountainous environment. It is important to note that the inventory of freeways used in this paper did not include any freeways which exceed volumes of 1,800 vphpl. This may explain why the reduction in crash rates associated with heavy congestion described by Kononov (5) is not reflected in the functional form of corridor specific SPFs in this study. Further, the limited range of speeds represented prevents detailed analysis of the way in which speed enters the threshold inequality. Figures 3-7 suggest that crash rate remains relatively stable until a certain threshold value of dS^2 is reached. Once it is exceeded, however, the crash rate begins to rise rapidly. Density (d) times speed squared (S^2) can be viewed as corridor specific Flow Crash Potential Indicator (FCPI), which reflects crash probability for different operational regimes.

$$FCPI=dS^2$$

The relationship between dS^2 (FCPI) and crash rates seems to resemble *a phase change phenomenon in chemistry or critical mass in physics.* A possible explanation may be that if FCPI exceeds a certain critical threshold value available headway becomes too small for the prevailing speed to allow drivers to react effectively to changing traffic conditions. Furthermore two (2) distinct operational regimes can be observed on Figure 8, as well all other corridor specific SPFs. Regime-1 where $FCPI<FCPI_{cr}$ and Regime-2 where $FCPI>FCPI_{cr}$.

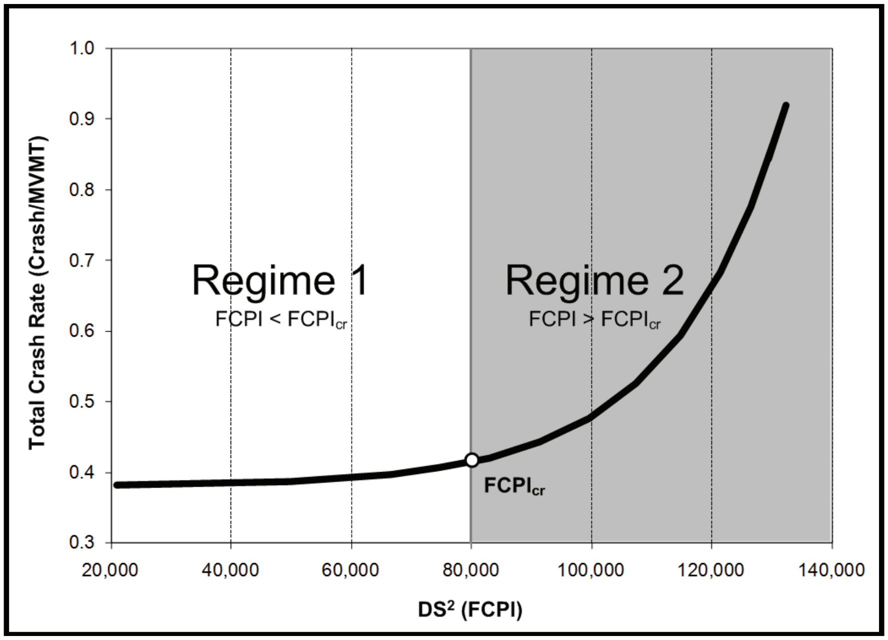

Figure 8 Corridor-specific SPF with Regimes 1 and 2

Regime 1 is characterized by low to moderate density and high speeds, where drivers are still able to compensate for increasing density. Increased focus on the driving task may possibly explain the fact that during Regime 1 the crash rate remains stable despite increase in density. Regime 2 is characterized by moderate to high densities without notable speed reduction where the combination of speed and density is such that more drivers are not able to compensate for driver's error and avoid a crash. In Regime 2 greater portion of near misses becomes crashes reflected by a sharp rise in the crash rate.

A possible strategy to counteract the deficit of available deceleration distance associated with a mix of high speeds and short headways is to slow traffic down in real time via Variable Speed Limits (VSL).

VARIABLE SPEED LIMITS (VSL) ALGORITHM

Variable speed limit control is an Active Traffic Management (ATM) strategy intended to maximize throughput, improve safety and reduce travel time variability. According to Chang et al., (11) a VSL system typically consists of a set of traffic sensors to collect flow and speed data, several properly located variable message signs (VMS) for message display, reliable control algorithm to compute the optimal speed for all control locations, and a real-time database as well as a communication system to convey information between all principal modules. The core of VSL logic (Chang et al.) is to dynamically adjust a set of speed limits to harmonize the speed transition between the upstream free-flow and downstream congested traffic states. This harmonizing or smoothing of traffic flow is thought to prevent the formation of excessive queue due to shock-wave effect. Hegyi, De Schutter and Hellendoorn (12) demonstrated that VSL can be an effective strategy to increase throughput on recurrently congested European freeways by reducing or eliminating the shock-wave. The principal aim of extensive VSL deployment in Europe was to improve safety and traffic operations on freeways. In contrast to our European counterpart's expertise, the state of reliable knowledge on safety and mobility benefits of VSL in the United States is emerging, but is limited at present. Golub et al. (13) identified flow patterns associated with crash types by using loop detectors in California and developed a software tool for predicting crash types most likely to occur. Substantive and innovative work in the general area of active traffic management and VSL in particular was done by Abdel-Aty et. al (14). Using a logistic regression model Abdel-Aty et al. have shown that high variability in speed observed 5-10 minutes before the crash represented by its coefficient of variation (=standard deviation/mean) was the most significant crash predictor. By the time speed variability is observed, however, it may be more difficult to effectively influence the flow by slowing it down. While speed variability is strongly correlated with crashes it may be more effective to intervene via VSL in advance of observing turbulence reflected by speed differential.

Figures 3-7 suggest that when a product of density times speed squared exceeds certain *corridor-specific* threshold or critical FCPI we begin to observe rapid deterioration of safety demonstrated by a rise in the crash rate. The critical value of FCPI can be estimated using *a sliding interval analysis in the framework of the numerical differentiation technique* described by Rao (15). A possible strategy to counteract the deficit of available deceleration distance associated with a mix of high speeds and short headways is to slow traffic down in real time via VSL. This idea lends itself to a following conceptual algorithm (Figure 8), where:

d_o – Observed Density of Flow (pcpmpl)

S_o – Observed Speed

$FCPI_o$- Observed Flow Crash Potential Index ($FFCPI_o = d_o S_o^2$)

$FCPI_{cr}$ – Critical corridor specific value of Freeway Flow Crash Potential Index estimated using corridor specific Safety Performance Function

S_r - Recommended Speed ($S_r = \sqrt{\frac{FCPI_{cr}}{d_0}}$) rounded to the nearest 5 mph

Ideally we would like to operate freeways in Regime-1 at less than critical values of FCPI, however a final resulting operating speed will be influenced by the degree of compliance. This conceptual algorithm is intended to compute recommended baseline speed on individual segments for which the SPF has been calibrated. In practice, the final VSL display will be informed by the real time traffic operations upstream and downstream. Figure 9 illustrates how the algorithm is intended to work by combining the corridor specific SPF with observed and recommended traffic flow parameters for a freeway with FCPI=80,000 and static speed limit of 70mph. Table 1 contains all related calculations and observed as well as recommended speeds, based on the hypothesized form of the threshold inequality.

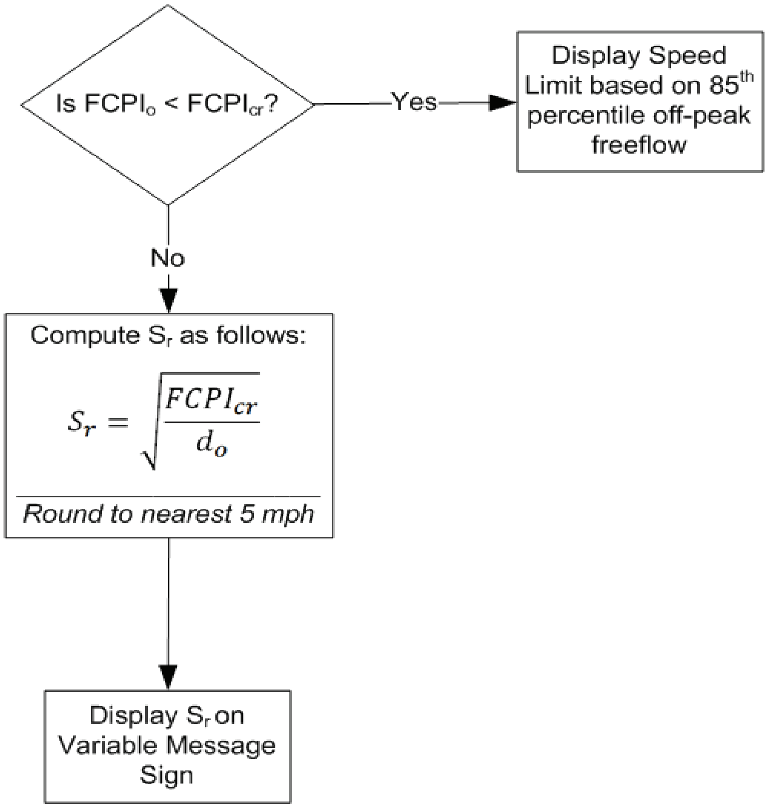

Figure 9 VSL Algorithm

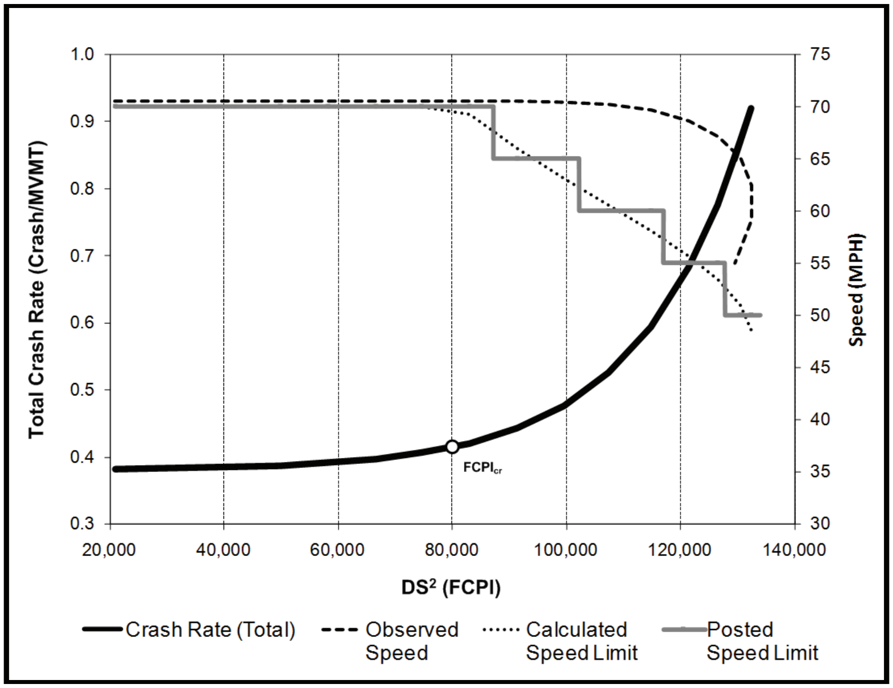

Figure 10 Corridor Specific SPF (FCPI=80,000) with Observed and Posted Speeds

Speed Observed (S)	***Density Observed*** (d_o)	***FCPI*** (d_oS^2)	$S_r = \sqrt{\frac{80,000}{d_o}}$	S_r *Rounded to nearest 5 mph Displayed on VMS*
70.5	4.2	20,875	FCPI < FCPI$_{cr}$ Display posted speed limit	70
70.5	18.4	91,453	65.9	65
70.4	20.1	99,619	63.1	65
70.2	21.8	107,431	60.6	60
69.6	23.7	114,807	58.1	60
68.6	25.8	121,414	55.7	55
67.1	28.1	126,518	53.4	55
65.1	30.8	130,531	51.0	50
62.4	34.0	132,388	48.5	50
59.1	37.9	132,377	45.9	45
55.0	42.8	129,470	43.2	45

Table 1 Observed Speeds and Recommended/Posted Speeds

Inclement weather adversely impacts safety as well traffic operations. Though speed-flow curves for snowy and rainy conditions are provided in the HCM (1), however the impact of adverse weather on freeway safety has not been fully calibrated. Preliminary results from the I-70 corridor used in this study suggest that crash rates computed for hourly volumes during ski season are notably higher than crash rates for the same volumes in the summer time. When the weather is a factor it is important to calibrate seasonally adjusted, corridor specific SPF to identify $FCPI_{cr}$. Figure 10 shows a decision tree reflecting the process of establishing VSL based on time of day and weather conditions.

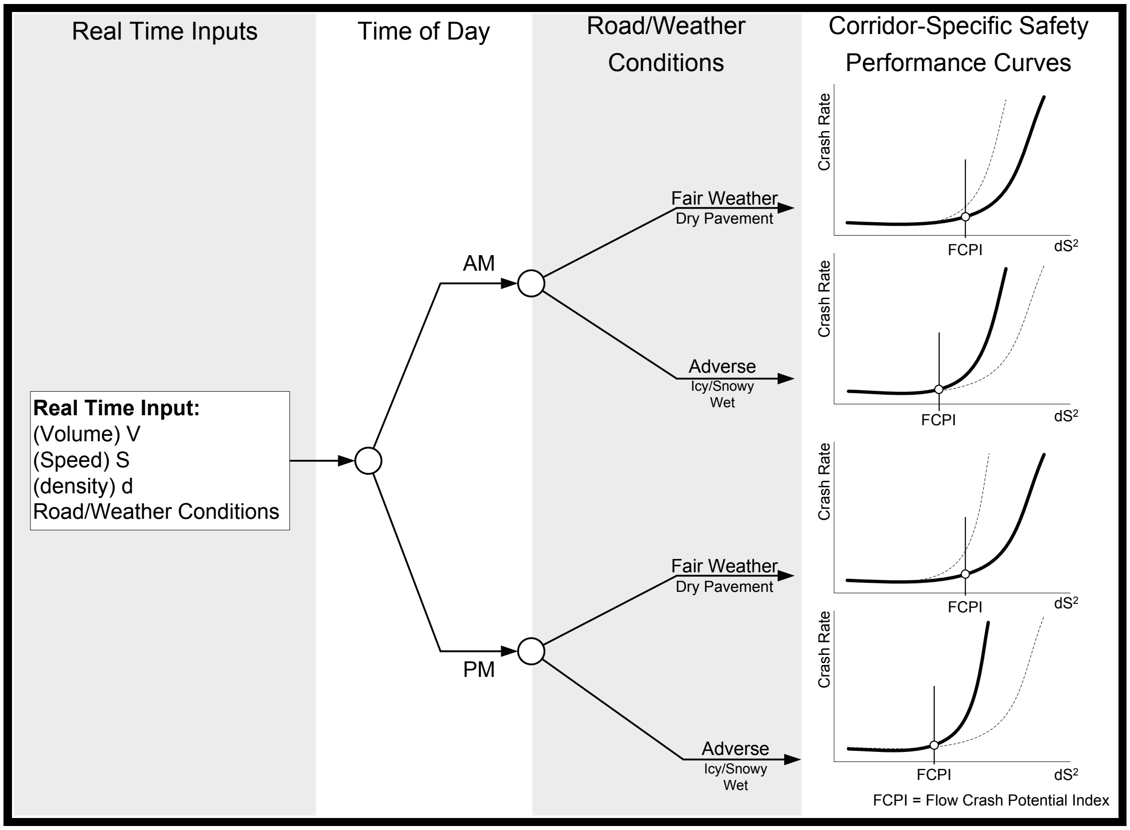

Figure 11 VSL Decision Tree

SUMMARY

All recent freeway studies show that speed on freeways is insensitive to flow in the low to mid range. Increase in flow and density without notable reduction in speed has a significant influence on safety. This influence, however, has not been studied extensively and has attracted only limited interest from researchers to date. Empirical examination of safety performance of Colorado freeways as a function of density times speed squared suggests that the crash rate remains relatively stable until a certain threshold is reached. The relationship between dS^2 or Flow Crash Potential Indicator (FCPI) and crash rates seems to resemble a critical mass-like phenomenon in physics. A possible explanation may be that if FCPI exceeds a certain critical threshold value available headway becomes too small to allow drivers traveling the prevailing speed to react effectively to changing traffic conditions. Relating basic kinematics with flow theory shows this interpretation to be consistent with a threshold based on the value of density times speed squared. Further empirical investigation over a wider range of speeds will be necessary to refine the relationship between speed, density, and the threshold. Two distinct operational regimes can be observed in all corridor-specific SPFs, Regime-1 where FCPI<FCPIcr and Regime-2 where FCPI>FCPIcr. Regime-1 is characterized by low to moderate density and high speeds, where drivers are becoming more focused on the driving task and are still able to compensate for increasing density. This increased focus on the driving task may possibly explain the fact that in Regime-1 the crash rate remains stable despite increase in density. Regime-2 is characterized by moderate to high densities without notable speed reduction where combination of speed and densities are such that many more near misses become crashes, thus a sharp rise in crash rate.

A possible strategy to counteract the deficit of available deceleration distance produced by a mix of high speeds and short headways is to slow traffic down in real time via Variable Speed Limit (VSL). A conceptual VSL algorithm proposed in this paper is intended to establish recommended baseline speed on individual freeway segments for which SPF has been calibrated. The final VSL display will be informed by real time traffic operations considerations. Deployment of such an algorithm has the potential to improve safety and reduce travel time variability. Additionally, underlying relationships between safety, speed and density of freeway flow have the potential to be integrated with various traffic simulation software packages currently in use.

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